

THE CHAMBER GRAPH OF THE M_{24} MAXIMAL 2-LOCAL GEOMETRY

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Abstract

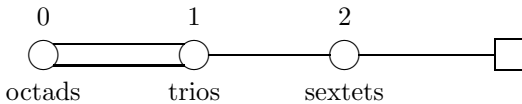
The chamber graph of the maximal 2-local geometry for M_{24} , the Mathieu group of degree 24, is analysed extensively. In addition to determining the discs around a fixed chamber of the chamber graph, the geodesic closure of an opposite pair of chambers is investigated.

1. *Introduction*

By a geometry (over the set I) we mean a triple $(\Gamma, \tau, *)$ where Γ is a set, τ an onto map from Γ to I and $*$ is a symmetric relation on Γ with the property that for $x, y \in \Gamma$, $x * y$ implies $\tau(x) \neq \tau(y)$. We refer to $*$ as the incidence relation and τ as the type map of the geometry; $x \in \Gamma$ is said to have type i if $\tau(x) = i$. For brevity, we write Γ to stand for $(\Gamma, \tau, *)$. A flag F is a set of pairwise incident elements of Γ . The rank of Γ is $|I|$ and the rank of F is $|\{\tau(x) \mid x \in F\}|$. Let \mathcal{C} denote the set of maximal flags (or chambers) of Γ – a flag F is maximal provided its rank is $|I|$. The chamber graph of Γ has \mathcal{C} as its vertex set and two (distinct) chambers F_1 and F_2 are adjacent whenever the rank of $F_1 \cap F_2$ is $|I| - 1$. We will denote the chamber graph of Γ by \mathcal{C} .

For further discussion on chamber systems see [7, 9] and for background on geometries consult [1].

Here we will be solely concerned with elucidating the combinatorial structure and uncovering interesting properties of the chamber graph of the M_{24} maximal 2-local geometry. This geometry first saw the light of day in [6], accompanied by the following diagram.



The square box is sometimes called a ghost node. There are no ghouls in this paper – we shall regard this as a rank 3-geometry, henceforth denoted Γ . The elements of Γ are the octads, trios and sextets of the Steiner system $S(24, 8, 5)$, with octads, trios and sextets having, respectively, type 0, 1 and 2, where $I = \{0, 1, 2\}$. An octad is deemed to be incident with a trio if it is one of the three octads of the trio, and to be incident with a sextet if it may be obtained as the union of two of the tetrads of the sextet. A trio and a sextet are incident if the three octads of the trio are unions of the tetrads of the sextet.

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So, at a more down to earth level, an element (chamber) of \mathcal{C} consists of an octad, trio and sextet such that the trio may be obtained from the sextet by a suitable pairing of the tetrads and the octad is one of the octads of the trio. Throughout this paper G denotes the copy of M_{24} preserving the Steiner system determined by Curtis's standard MOG array [4]. We recall that G is a subgroup of $\text{Aut } \Gamma$ which acts transitively on the chambers of Γ (that is the vertices of \mathcal{C}).

Apart from anything to do with M_{24} being of intrinsic interest, it is the geometric context that motivates much of our investigations. In the geometric world Γ occupies an exalted position by virtue of its appearance as a residue geometry in other geometries associated with $\cdot 1, M, Fi_{24}$ and J_4 (see [6] again). The class of buildings is also much revered in the geometry world. In fact, the chamber graph of a building encodes many of the building axioms and concepts (such as galleries, thin subgeometries – see [7, 8] for more on this). This then leads us to compare and contrast the chamber graph of other geometries with those of buildings.

We use $d(\cdot, \cdot)$ to denote the distance function on the chamber graph \mathcal{C} and, for $c \in \mathcal{C}$, define the j th disc of c to be

$$D_j(c) := \{c' \in \mathcal{C} \mid d(c, c') = j\}.$$

Observing that Γ has $1771 \cdot 15 \cdot 3 = 79,695$ chambers, we come to our first result.

THEOREM 1. *Let c_0 be a fixed chamber in \mathcal{C} . Then the sizes of $D_j(c_0)$ are as follows.*

j	0	1	2	3	4	5	6	7	8	9	10
$ D_j(c_0) $	1	10	44	184	544	1536	4800	10368	22272	38400	1536

Theorem 1 is a consequence of a much more detailed picture of \mathcal{C} which is unveiled in Section 3. Various other properties of \mathcal{C} also emerge; concerning the last disc $D_{10}(c_0)$ we mention

THEOREM 2. *Let c_0 be a fixed chamber in \mathcal{C} . Then G_{c_0} is transitive on $D_{10}(c_0)$ and, furthermore, $D_{10}(c_0)$ is a coclique.*

Theorem 2 leads us to view pairs of chambers which are distance 10 apart (in \mathcal{C}) with some interest. We shall call such a pair of chambers in \mathcal{C} an opposite pair of chambers.

For $c, c' \in \mathcal{C}$ a geodesic (or minimal gallery) from c to c' is a shortest path in the graph \mathcal{C} , starting with c and ending with c' . Let \mathcal{X} be a subset of \mathcal{C} . The geodesic closure of \mathcal{X} , denoted by $\bar{\mathcal{X}}$, consists of all the chambers in \mathcal{C} which lie on a geodesic between c and c' where $c, c' \in \mathcal{X}$. The geodesic closure of two chambers that are maximal distance apart in the chamber graph of a building of spherical type yields (the chambers of) an apartment and every apartment of the building appears in this way (see [6; 2.15 Theorem and 3.8 Theorem]). This leads us to the subject of our next result.

THEOREM 3. *Suppose $\{c_0, c_{10}\}$ is an opposite pair of chambers in \mathcal{C} , and set $\Delta_j = \overline{\{c_0, c_{10}\}} \cap D_j(c_0)$. Then the sizes of Δ_j are as follows.*

j	0	1	2	3	4	5	6	7	8	9	10
$ \Delta_j $	1	10	12	14	15	14	15	14	12	10	1

In Section 4 we scrutinize $\overline{\{c_0, c_{10}\}}$ in some detail; indeed in an appendix we give a listing of all 118 chambers in $\overline{\{c_0, c_{10}\}}$ for a specific c_0 and c_{10} since we

believe/hope this set of chambers will repay further study. We note that the ‘symmetry’ of the sizes in Theorem 3 is a consequence of G being transitive on \mathcal{C} and $D_{10}(c_0)$ being a G_{c_0} -orbit. Further properties and a discussion of this graph may be found in [8].

Briefly, this paper is arranged as follows. In the next section, we review properties of the Steiner system $S(24, 8, 5)$ with, as might be expected, particular emphasis on the octads, trios and sextets. We employ Curtis’s MOG [4] both as our main descriptive device and also as an essential tool in our calculations. Accordingly, in Section 2, we spruce up MOG ready for action in Section 3 where we determine the discs $D_j(c_0)$ for a fixed $c_0 \in \mathcal{C}$. As mentioned above, Section 4 examines the geodesic closure of a pair of opposite chambers in \mathcal{C} , as well as airing some questions about \mathcal{C} , with further details given in Appendix A and Appendix B. Finally, Appendix C gives some MAGMA code (supplied by the referee who we thank) which calculates the lengths of the G_{c_0} -orbits on \mathcal{C} . This is to act as a check for some of the data in (3.2).

2. Notation and CHAMBERMOG

Let Ω be a 24-element set which we assume comes equipped with the Steiner system $S(24, 8, 5)$. We denote the set of all octads, trios and sextets of Ω by, respectively, \mathcal{O} , \mathcal{T} and \mathcal{S} .

For $c \in \mathcal{C}$, we use, respectively, $O(c)$, $T(c)$ and $S(c)$ to denote the octad, trio and sextet of c . So $T(c)$ is obtained via a suitable partition of the tetrads of $S(c)$ and $O(c)$ is one of the three octads of $T(c)$. The three heavy blocks of the MOG are labelled thus

$$\Omega = \left[\begin{array}{|c|c|c|} \hline O_1 & O_2 & O_3 \\ \hline \end{array} \right].$$

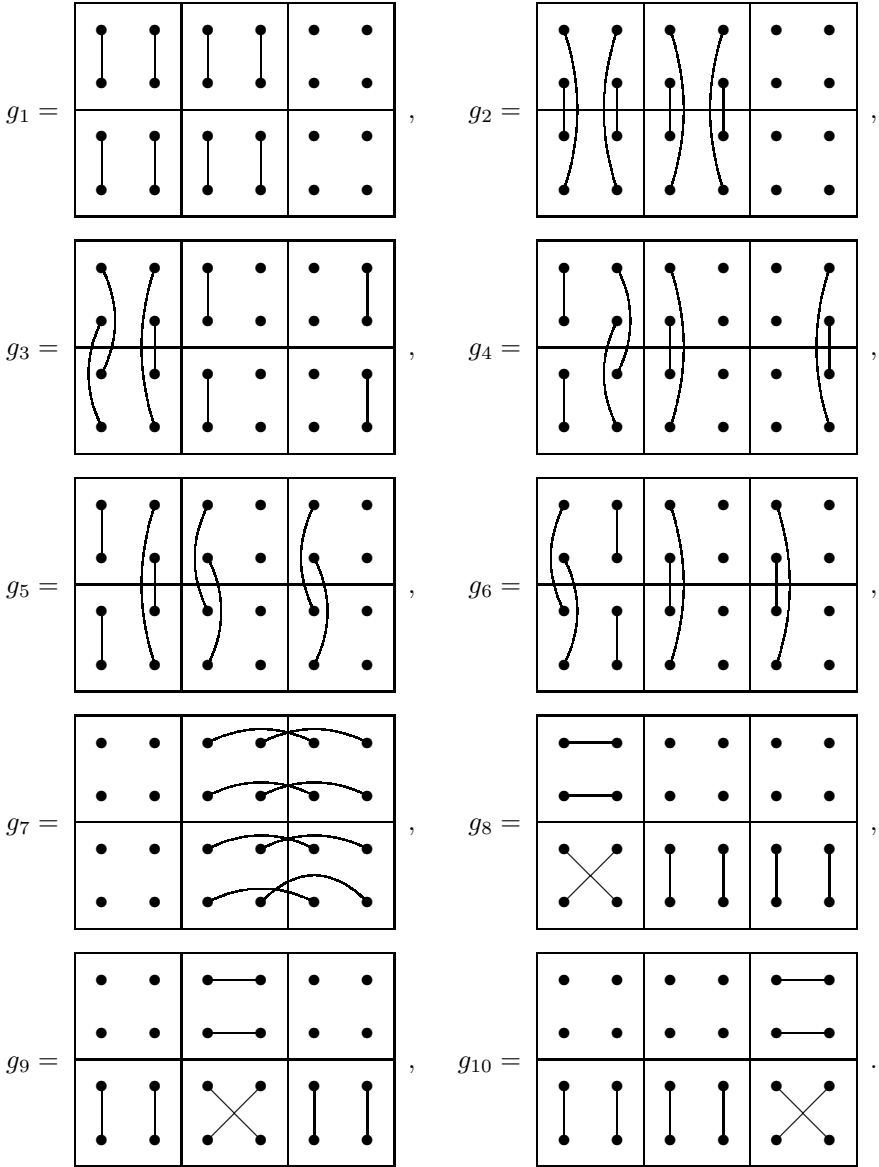
By the terms standard trio and standard sextet we mean the following trio and sextet.

$$T_0 := \left[\begin{array}{|c|c|c|} \hline + + & - - & \circ \circ \\ + + & - - & \circ \circ \\ + + & - - & \circ \circ \\ + + & - - & \circ \circ \\ \hline \end{array} \right] \quad S_0 := \left[\begin{array}{|c|c|c|} \hline + - & \times * & \circ \diamond \\ + - & \times * & \circ \diamond \\ + - & \times * & \circ \diamond \\ + - & \times * & \circ \diamond \\ \hline \end{array} \right]$$

Throughout this paper, c_0 denotes the chamber of \mathcal{C} for which $O(c_0) = O_1$, $T(c_0) = T_0$ and $S(c_0) = S_0$. Let B denote the stabilizer in $G \cong M_{24}$ of c_0 . Then $B = G_{O_1} \cap G_{T_0} \cap G_{S_0}$ has shape $2^6 : (3 \times D_8) : 2$ and $B = MN_B(\langle \rho \rangle)$ where

$$\rho = \left[\begin{array}{|c|c|c|} \hline \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \hline \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \hline \end{array} \right]$$

and $M = \langle g_i \mid 1 \leq i \leq 6 \rangle$ and $N_B(\langle \rho \rangle) = \langle \rho, g_7, g_8, g_9, g_{10} \rangle$ with



We note that $B = N_G(\langle \rho \rangle)$ is the stabilizer of the top row of the MOG and is isomorphic to the triple cover $3 \cdot S_6$. Further, M is an elementary abelian group of order 2^6 whose elements may also be labelled by the words of the hexacode. If $\{0, 1, \omega, \bar{\omega}\}$ are the elements of the Galois field $GF(4)$, then 0 corresponds to the identity (on a column), 1 corresponds to interchanging the 1st and 2nd, 3rd and 4th entries, ω corresponds to interchanging the 1st and 3rd, 2nd and 4th entries and $\bar{\omega}$ corresponds to interchanging the 1st and 4th, 2nd and 3rd entries. So g_4 corresponds to the hexacode word $(1, \omega, \bar{\omega}, 0, 0, \bar{\omega})$. See the M_{24} page in [2] for further details.

For distinct sextets X_1 and X_2 , we have the following three possibilities for their

intersection matrix. (The (i, j) th entry is the number of elements in the intersection of the i th tetrad of X_1 with the j th tetrad of X_2 , given a labelling of the tetrads of X_1 and X_2 .)

$$\mathcal{S}_0 = \begin{pmatrix} 2 & & & & 1 & 1 \\ & 2 & & & 1 & 1 \\ & & 2 & & 1 & 1 \\ & & & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & & \\ 1 & 1 & 1 & 1 & & \end{pmatrix} \quad \mathcal{S}_1 = \begin{pmatrix} 3 & 1 & & & & \\ 1 & 3 & & & & \\ & & 1 & 1 & 1 & 1 \\ & & 1 & 1 & 1 & 1 \\ & & 1 & 1 & 1 & 1 \\ & & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\mathcal{S}_3 = \begin{pmatrix} 2 & 2 & & & & \\ 2 & 2 & & & & \\ & & 2 & 2 & & \\ & & 2 & 2 & & \\ & & & & 2 & 2 \\ & & & & 2 & 2 \end{pmatrix}$$

For a fixed sextet X let $\sigma_i(X)$ denote the set of sextets which have an \mathcal{S}_i intersection matrix with X , $i = 0, 1, 3$. (So $\mathcal{S} = \{X\} \cup \sigma_0(X) \cup \sigma_1(X) \cup \sigma_3(X)$.) When $X = S_0$ we will write σ_i in place of $\sigma_i(X)$. Note that for $X_1 \in \sigma_i(X)$, X_1 and X have precisely i octads in common.

(2.1) Let X be a fixed sextet. Then

- (i) $|\sigma_0(X)| = 1440$, $|\sigma_1(X)| = 240$, $|\sigma_3(X)| = 90$; and
- (ii) the G_X -orbits on the set of sextets are X , $\sigma_0(X)$, $\sigma_1(X)$, $\sigma_3(X)$.

Proof. See [3, Chapter 1, Section 2]. □

Next we consider trios. Let Y_1 and Y_2 be distinct trios. Then there are four possible ways their octads can intersect. Here the (i, j) th entry is the size of the intersection of the i th octad of Y_1 with the j th octad of Y_2 , assuming a labelling of the octads of Y_1 and Y_2 .

$$\mathcal{T}_0 = \begin{pmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{pmatrix} \quad \mathcal{T}_1 = \begin{pmatrix} 2 & 2 & 4 \\ 2 & 2 & 4 \\ 4 & 4 & 0 \end{pmatrix}$$

$$\mathcal{T}_2 = \begin{pmatrix} 8 & 0 & 0 \\ 0 & 4 & 4 \\ 0 & 4 & 4 \end{pmatrix} \quad \mathcal{T}_3 = \begin{pmatrix} 4 & 4 & 0 \\ 4 & 0 & 4 \\ 0 & 4 & 4 \end{pmatrix}$$

For a trio Y , $\tau_i(Y)$ will denote the set of trios whose octad intersection matrix with Y is \mathcal{T}_i , $i = 0, 1, 2, 3$. In the case when $Y = T_0$, we shorten $\tau_i(Y)$ to τ_i .

We usually describe a sextet X in the following manner. For $i \in \{1, \dots, 6\}$ the 4-element subset of the MOG labelled by i defines the tetrads of X . So

$$S_0 := \begin{array}{|c|c|c|} \hline 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 2 & 3 & 4 & 5 & 6 \\ \hline \end{array}.$$

Also note that, for example,

$$\begin{array}{|c|c|c|c|} \hline 2 & 1 & 3 & 3 \\ \hline 1 & 2 & 4 & 4 \\ \hline 1 & 2 & 5 & 5 \\ \hline 1 & 2 & 6 & 6 \\ \hline \end{array} \quad \text{and} \quad \begin{array}{|c|c|c|c|} \hline 5 & 4 & 1 & 1 \\ \hline 4 & 5 & 2 & 2 \\ \hline 4 & 5 & 3 & 3 \\ \hline 4 & 5 & 6 & 6 \\ \hline \end{array}$$

describe the same sextet, say X_1 .

We employ two schemes for describing chambers which we explain with an example. By

$$c_1 := \begin{array}{|c|c|c|c|} \hline 2 & 1 & 3 & 3 \\ \hline 1 & 2 & 4 & 4 \\ \hline 1 & 2 & 5 & 5 \\ \hline 1 & 2 & 6 & 6 \\ \hline \end{array} \quad \text{or} \quad c_1 := \begin{array}{|c|c|c|c|} \hline 2 & 1 & 3^+ & 3 \\ \hline 1 & 2 & 4^+ & 4 \\ \hline 1 & 2 & 5^- & 5 \\ \hline 1 & 2 & 6^- & 6 \\ \hline \end{array}$$

12|34|56

we mean that c_1 is a chamber with $O(c_1) = O_1$, $T(c_1) = T_0$ and $S(c_1) = X_1$. The 12|34|56 tells us how to partition the tetrads to obtain the trio and the underlined partition gives the octad of the chamber. In the alternative notation the + on the 3 and 4 and the - on the 5 and 6 indicate two of the octads of the trio and the ‘unmarked’ octad is the octad of the chamber. Dressed in this way MOG becomes CHAMBERMOG.

3. B -orbits and disc structure

Our strategy for determining the chambers in a particular $D_j(c_0)$ is first to determine the B -orbits of \mathcal{C} . Clearly, if a chamber c is in $D_j(c_0)$, then the B -orbit of c will be contained in $D_j(c_0)$. Beginning with c_0 we work outwards building up successive discs of c_0 . Suppose we have successfully enumerated $D_i(c_0)$ for $i \leq j$. Taking $c \in D_j(c_0)$ we calculate, using the MOG, the ten chambers of \mathcal{C} adjacent to c . In the light of our knowledge of $D_i(c_0)$, $i \leq j$, we discover which of the ten chambers are in $D_{j+1}(c_0)$, and hence the B -orbits of these chambers will be in $D_{j+1}(c_0)$. Letting c run through representatives of the B -orbits of $D_j(c_0)$, this procedure delivers $D_{j+1}(c_0)$ as a union of B -orbits. And we repeat this procedure until we run out of chambers!

We arrive at the orbits of B on the chambers of \mathcal{C} in a series of stages. First we determine the orbits of B upon the sextets of Ω . Then taking a representative sextet X from each of these we may obtain the 15 trios incident with X whence, by nominating a particular octad, we then get the 45 chambers which contain X . Examining the action of B_X and combinatorial properties of such chambers in relation to c_0 allows us to compile the B -orbits of \mathcal{C} .

In order to describe the B -orbits of \mathcal{S} we need some auxiliary notions.

Let $X \in \mathcal{S}$, and first we suppose that $X \in \sigma_0$. Then there will be precisely two columns (of the MOG) and four tetrads of X each of which intersect the two columns (of the MOG) in one element. We shall refer to either of these two columns

of the MOG as mixed cols of X . So, for example, if

$$X = \begin{array}{|c|c|c|} \hline 6 & 4 & 3 & 2 & 5 & 4 \\ \hline 3 & 3 & 5 & 1 & 5 & 6 \\ \hline 5 & 1 & 2 & 2 & 1 & 6 \\ \hline 2 & 3 & 6 & 4 & 4 & 1 \\ \hline \end{array},$$

then the first and third columns of the MOG are the mixed cols of X . Now consider the case when $X \in \sigma_1$. Here we have exactly two columns of the MOG for which two of the tetrads of X intersect these columns in 3 elements. Such columns of the MOG will be called 3-cols of X . If, say,

$$X = \begin{array}{|c|c|c|c|} \hline 3 & 3 & 3 & 3 & 1 & 2 \\ \hline 4 & 4 & 4 & 4 & 2 & 1 \\ \hline 5 & 5 & 5 & 5 & 2 & 1 \\ \hline 6 & 6 & 6 & 6 & 2 & 1 \\ \hline \end{array},$$

then the fifth and sixth columns of the MOG are the 3-cols of X . Lastly we look at $X \in \sigma_3$. In this case the six columns of the MOG are partitioned into three pairs where each pair of columns is determined by two tetrads of X each of which intersects both columns of the pair in two elements. Any three of these pairs of columns we call a col pair of X . If, for example,

$$X = \begin{array}{|c|c|c|c|} \hline 5 & 6 & 2 & 3 & 4 & 1 \\ \hline 6 & 5 & 2 & 3 & 4 & 1 \\ \hline 6 & 6 & 4 & 1 & 2 & 3 \\ \hline 5 & 5 & 4 & 1 & 2 & 3 \\ \hline \end{array},$$

then the col pairs of X are columns 1 and 2, columns 3 and 5, and columns 4 and 6.

Since $B \leq G_{S_0}$, we see that σ_i ($i = 0, 1, 3$) will be a union of B -orbits. Hence we label the B -orbits of \mathcal{S} by $\sigma_i^{(n)}$ to indicate that $\sigma_i^{(n)} \subseteq \sigma_i$ ($i = 0, 1, 3$); the n is the size of that particular B -orbit.

(3.1) B has 12-orbits on \mathcal{S} the details of which are tabulated below.

B -ORBIT	SIZE	DESCRIPTION
$\{S_0\}$	1	standard sextet
$\sigma_0^{(96)}$	96	both mixed cols in O_1
$\sigma_0^{(192)}$	192	both mixed cols either in O_2 or in O_3
$\sigma_0^{(384)}$	384	one mixed col in O_2 , one mixed col in O_3
$\sigma_0^{(768)}$	768	one mixed col in O_1 , one mixed col either in O_2 or in O_3
$\sigma_1^{(16)}$	16	both 3-cols in O_1
$\sigma_1^{(32)}$	32	both 3-cols either in O_2 or in O_3
$\sigma_1^{(64)}$	64	one 3-col in O_2 , one 3-col in O_3
$\sigma_1^{(128)}$	128	one 3-col in O_1 , one 3-col either in O_2 or in O_3
$\sigma_3^{(6)}$	6	each col pair contained in one of O_1, O_2, O_3
$\sigma_3^{(12)}$	12	one col pair in O_1 , no col pairs either in O_2 or in O_3
$\sigma_3^{(24)}$	24	one col pair in O_2 , no col pairs either in O_1 or in O_3 <u>or</u> one col pair in O_3 , no col pair either in O_1 or in O_2
$\sigma_3^{(48)}$	48	no col pairs in any of O_1, O_2, O_3

Building upon the information in (3.1) we obtain the B -orbits of \mathcal{C} and then we implement the procedure discussed earlier, the results of which, together with the B -orbits of \mathcal{C} , are presented in (3.2). After (3.2) we explain how the MOG helps us to calculate the adjacent chambers of a given chamber. In columns 2, 3, 4 and 5 of the tables in (3.2) we give, respectively, the sizes of a B -orbit, the j for which $D_j(c_0)$ contains the B -orbit, τ_i for which $T(c) \in \tau_i$ and $|O(c) \cap O_1|$ (where c is a chamber in the B -orbit).

(3.2)

$S(c) = S_0:$

$$S(c) = \begin{array}{|c|c|c|} \hline 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 2 & 3 & 4 & 5 & 6 \\ \hline \end{array},$$

B -orbit representative	size	disc	τ_i	O_1 -intersection
<u>12</u> 34 56	1	0	τ_0	8
12 <u>34</u> 56	2	1	τ_0	0
<u>12</u> 35 46	2	1	τ_2	8
15 26 <u>34</u>	4	2	τ_2	0
12 <u>35</u> 46	4	2	τ_2	0
<u>15</u> 26 34	8	3	τ_2	4
14 26 <u>35</u>	8	3	τ_3	0
<u>13</u> 25 46	16	4	τ_3	4

$S(c) \in \sigma_0^{(96)}:$

$$S(c) = \begin{array}{|c|c|c|} \hline 1 & 3 & 5 & 1 & 3 & 5 \\ \hline 2 & 4 & 6 & 1 & 3 & 6 \\ \hline 3 & 2 & 2 & 5 & 5 & 4 \\ \hline 4 & 1 & 2 & 6 & 6 & 4 \\ \hline \end{array},$$

B -orbit representative	size	disc	τ_i	O_1 -intersection
12 34 <u>56</u>	1×96	5	τ_3	0
13 24 <u>56</u>	2×96	6	τ_1	0
<u>12</u> 34 56	2×96	6	τ_3	4
<u>13</u> 24 56	4×96	7	τ_1	4
<u>12</u> 35 46	4×96	7	τ_1	4
<u>13</u> 25 46	8×96	8	τ_0	4
<u>15</u> 26 34	8×96	8	τ_1	2
<u>15</u> 23 46	16×96	9	τ_0	2

$S(c) \in \sigma_0^{(192)}:$

$$S(c) = \begin{array}{|c|c|c|} \hline 5 & 1 & 1 & 3 & 3 & 5 \\ \hline 6 & 1 & 2 & 4 & 3 & 6 \\ \hline 2 & 5 & 3 & 2 & 5 & 4 \\ \hline 2 & 6 & 4 & 1 & 6 & 4 \\ \hline \end{array},$$

B -orbit representative	size	disc	τ_i	O_1 -intersection
12 <u>34</u> 56	1×192	5	τ_3	0
15 <u>34</u> 26	2×192	6	τ_1	0
12 34 <u>56</u>	1×192	6	τ_3	4
<u>12</u> 34 56	1×192	6	τ_3	4
<u>16</u> 25 34	4×192	7	τ_1	4
<u>12</u> 35 46	2×192	7	τ_1	4
13 24 <u>56</u>	2×192	7	τ_1	4
<u>15</u> 23 46	8×192	8	τ_0	4
12 <u>35</u> 46	4×192	8	τ_1	2
<u>13</u> 24 56	4×192	8	τ_1	2
14 26 <u>35</u>	8×192	9	τ_0	2
<u>13</u> 25 46	8×192	9	τ_0	2

The chamber graph of the M_{24} maximal 2-local geometry

$S(c) \in \sigma_0^{(384)}$:

$$S(c) = \begin{array}{|c|c|c|} \hline 5 & 3 & 1 & 6 & 3 & 1 \\ \hline 4 & 5 & 2 & 5 & 4 & 1 \\ \hline 4 & 3 & 3 & 2 & 2 & 6 \\ \hline 6 & 6 & 4 & 2 & 1 & 5 \\ \hline \end{array},$$

B -orbit representative	size	disc	τ_i	O_1 -intersection
<u>12</u> <u>34</u> 56	1×384	6	τ_1	0
<u>12</u> <u>35</u> 46	2×384	6	τ_1	0
12 <u>35</u> 46	4×384	7	τ_1	4
12 <u>34</u> 56	1×384	7	τ_1	4
12 34 <u>56</u>	1×384	7	τ_1	4
14 <u>35</u> 26	8×384	8	τ_0	4
15 <u>26</u> <u>34</u>	2×384	8	τ_0	4
14 <u>56</u> 23	2×384	8	τ_0	4
<u>13</u> 25 46	8×384	9	τ_0	2
14 <u>25</u> 36	8×384	9	τ_0	2
<u>16</u> 25 34	4×384	9	τ_0	2
14 <u>23</u> 56	4×384	9	τ_0	2

$S(c) \in \sigma_0^{(768)}$:

$$S(c) = \begin{array}{|c|c|c|} \hline 6 & 4 & 3 & 2 & 5 & 4 \\ \hline 3 & 3 & 5 & 1 & 5 & 6 \\ \hline 5 & 1 & 2 & 2 & 1 & 6 \\ \hline 2 & 3 & 6 & 4 & 4 & 1 \\ \hline \end{array},$$

B -orbit representative	size	disc	τ_i	O_1 -intersection
14 <u>23</u> 56	1×768	7	τ_1	4
15 <u>23</u> 46	2×768	7	τ_1	4
<u>14</u> 23 56	1×768	8	τ_1	2
14 23 <u>56</u>	1×768	8	τ_1	2
<u>15</u> 23 46	4×768	8	τ_1	2
12 <u>35</u> 46	4×768	8	τ_0	4
14 <u>26</u> <u>35</u>	2×768	8	τ_0	4
<u>13</u> 24 56	2×768	9	τ_0	4
<u>13</u> 25 46	4×768	9	τ_0	4
<u>14</u> 25 36	2×768	9	τ_0	2
12 34 <u>56</u>	2×768	9	τ_0	2
12 36 <u>45</u>	4×768	9	τ_0	2
13 26 <u>45</u>	4×768	9	τ_0	2
<u>12</u> 36 45	4×768	9	τ_0	2
14 <u>25</u> 36	2×768	9	τ_0	2
13 <u>25</u> 46	4×768	9	τ_0	2
<u>12</u> 34 56	2×768	10	τ_0	2

$S(c) \in \sigma_1^{(16)}$:

$$S(c) = \begin{array}{|c|c|c|} \hline 2 & 1 & 3 & 3 & 3 & 3 \\ \hline 1 & 2 & 4 & 4 & 4 & 4 \\ \hline 1 & 2 & 5 & 5 & 5 & 5 \\ \hline 1 & 2 & 6 & 6 & 6 & 6 \\ \hline \end{array},$$

B -orbit representative	size	disc	τ_i	O_1 -intersection
<u>12</u> 34 56	3×16	3	τ_2	8
12 <u>34</u> 56	6×16	4	τ_2	0
15 26 <u>34</u>	12×16	5	τ_1	0
<u>13</u> 24 56	24×16	6	τ_1	4

$S(c) \in \sigma_1^{(32)}$:

$$S(c) = \begin{array}{|c|c|c|} \hline 3 & 3 & 2 & 1 & 3 & 3 \\ \hline 4 & 4 & 1 & 2 & 4 & 4 \\ \hline 5 & 5 & 1 & 2 & 5 & 5 \\ \hline 6 & 6 & 1 & 2 & 6 & 6 \\ \hline \end{array},$$

B -orbit representative	size	disc	τ_i	O_1 -intersection
<u>12</u> 34 56	3×32	4	τ_2	0
12 <u>34</u> 56	6×32	5	τ_2	4
13 24 <u>56</u>	12×32	6	τ_1	4
<u>13</u> 24 56	24×32	7	τ_1	2

$S(c) \in \sigma_1^{(64)}$:

$$S(c) = \begin{array}{|c|c|c|} \hline 5 & 4 & 1 & 3 & 1 & 6 \\ \hline 6 & 5 & 2 & 5 & 1 & 4 \\ \hline 4 & 6 & 2 & 4 & 1 & 5 \\ \hline 3 & 3 & 2 & 6 & 2 & 3 \\ \hline \end{array},$$

B -orbit representative	size	disc	τ_i	O_1 -intersection
<u>12</u> 34 56	3×64	5	τ_1	0
12 <u>34</u> 56	6×64	6	τ_1	4
13 24 <u>56</u>	12×64	7	τ_0	4
<u>13</u> 24 56	24×64	8	τ_0	2

$S(c) \in \sigma_1^{(128)}$:

$$S(c) = \begin{array}{|c|c|c|} \hline 1 & 4 & 5 & 3 & 1 & 6 \\ \hline 2 & 5 & 6 & 5 & 1 & 4 \\ \hline 2 & 6 & 4 & 4 & 1 & 5 \\ \hline 2 & 3 & 3 & 6 & 2 & 3 \\ \hline \end{array},$$

B -orbit representative	size	disc	τ_i	O_1 -intersection
<u>12</u> 34 56	3×128	6	τ_1	4
12 <u>34</u> 56	6×128	7	τ_1	2
13 24 <u>56</u>	12×128	8	τ_0	2
<u>13</u> 24 56	12×128	9	τ_0	2
14 <u>23</u> 56	12×128	9	τ_0	4

$S(c) \in \sigma_3^{(6)}$:

$$S(c) = \begin{array}{|c|c|c|} \hline 1 & 1 & 3 & 3 & 5 & 5 \\ \hline 1 & 1 & 3 & 3 & 5 & 5 \\ \hline 2 & 2 & 4 & 4 & 6 & 6 \\ \hline 2 & 2 & 4 & 4 & 6 & 6 \\ \hline \end{array},$$

B -orbit representative	size	disc	τ_i	O_1 -intersection
<u>12</u> 34 56	1×6	1	τ_0	8
12 <u>34</u> 56	2×6	2	τ_0	0
<u>12</u> 35 46	2×6	2	τ_2	8
12 <u>35</u> 46	4×6	3	τ_2	0
13 24 <u>56</u>	4×6	3	τ_2	0
13 26 <u>45</u>	8×6	4	τ_3	0
<u>13</u> 24 56	8×6	4	τ_2	4
<u>13</u> 25 46	16×6	5	τ_3	4

$S(c) \in \sigma_3^{(12)}$:

$$S(c) = \begin{array}{|c|c|c|} \hline 2 & 1 & 3 & 5 & 6 & 4 \\ \hline 1 & 2 & 3 & 5 & 6 & 4 \\ \hline 1 & 1 & 6 & 4 & 3 & 5 \\ \hline 2 & 2 & 6 & 4 & 3 & 5 \\ \hline \end{array},$$

B -orbit representative	size	disc	τ_i	O_1 -intersection
<u>12</u> <u>36</u> 45	1×12	2	τ_2	8
<u>12</u> <u>34</u> 56	2×12	3	τ_2	8
12 <u>36</u> 45	2×12	3	τ_2	0
12 <u>34</u> 56	4×12	4	τ_2	0
14 <u>25</u> <u>36</u>	4×12	4	τ_1	0
15 <u>26</u> <u>34</u>	8×12	5	τ_1	0
<u>14</u> <u>25</u> 36	8×12	5	τ_1	4
<u>14</u> <u>23</u> 56	16×12	6	τ_1	4

$S(c) \in \sigma_3^{(24)}$:

$$S(c) = \begin{array}{|c|c|c|} \hline 1 & 3 & 1 & 3 & 5 & 5 \\ \hline 1 & 3 & 1 & 3 & 6 & 6 \\ \hline 2 & 4 & 2 & 4 & 6 & 5 \\ \hline 2 & 4 & 2 & 4 & 5 & 6 \\ \hline \end{array},$$

B -orbit representative	size	disc	τ_i	O_1 -intersection
12 <u>34</u> <u>56</u>	1×24	3	τ_2	0
13 <u>24</u> <u>56</u>	2×24	4	τ_2	0
<u>12</u> <u>34</u> 56	2×24	4	τ_2	4
<u>13</u> <u>24</u> 56	4×24	5	τ_2	4
<u>12</u> <u>35</u> 46	4×24	5	τ_1	4
<u>13</u> <u>25</u> 46	8×24	6	τ_1	4
<u>15</u> <u>26</u> 34	8×24	6	τ_1	2
<u>15</u> <u>23</u> 46	16×24	7	τ_1	2

$S(c) \in \sigma_3^{(48)}$:

$$S(c) = \begin{array}{|c|c|c|} \hline 1 & 4 & 2 & 6 & 3 & 5 \\ \hline 2 & 3 & 2 & 5 & 3 & 6 \\ \hline 2 & 4 & 1 & 6 & 4 & 6 \\ \hline 1 & 3 & 1 & 5 & 4 & 5 \\ \hline \end{array},$$

B -orbit representative	size	disc	τ_i	O_1 -intersection
12 <u>34</u> <u>56</u>	1×48	4	τ_3	0
13 <u>24</u> <u>56</u>	2×48	5	τ_1	0
<u>12</u> <u>34</u> 56	2×48	5	τ_3	4
<u>13</u> <u>24</u> 56	4×48	6	τ_1	4
<u>12</u> <u>36</u> 45	4×48	6	τ_1	4
<u>13</u> <u>25</u> 46	8×48	7	τ_0	4
<u>15</u> <u>26</u> 34	8×48	7	τ_1	2
<u>15</u> <u>23</u> 46	16×48	8	τ_0	2

We illustrate the procedure adopted to calculate the neighbours of a chamber in the chamber graph, selecting

$$c_{10} := \begin{array}{|c|c|c|} \hline 6 & 4 & 3 & 2 & 5 & 4 \\ \hline 3 & 3 & 5 & 1 & 5 & 6 \\ \hline 5 & 1 & 2 & 2 & 1 & 6 \\ \hline 2 & 3 & 6 & 4 & 4 & 1 \\ \hline \end{array}$$

12|34|56

for demonstration purposes. Four of the ten neighbours of c_{10} are readily obtained where the sextet is kept the same and the varying of the octad and trio is given by 12|34|56, 12|34|56, 12|35|46 and 12|36|45. To locate the remaining six neighbours

of c_{10} (which will all have the same octad and trio as c_{10}), MOG comes out to play. Note that we are hunting for sextets whose tetrads are each contained in one of the octads of $T(c_{10})$ as well as hitting two tetrads of $S(c_{10})$ each in two elements. Let t_1, \dots, t_6 denote the six tetrads of $S(c_{10})$ and $O(1, 2) = t_1 \cup t_2$, $O(3, 4) = t_3 \cup t_4$, $O(5, 6) = t_5 \cup t_6$, the three octads of $T(c_{10})$.

First we select any 2-element subset of t_1 , say

$$D := \begin{array}{|c|c|c|} \hline & & \\ \hline & & * \\ \hline & & * \\ \hline \end{array}.$$

Now we look for an octad contained in $D \cup t_2 \cup t_3 \cup t_4$ and which contains D . Consulting the MOG gives

$$Oct_1 := \begin{array}{|c|c|c|} \hline & * & * * \\ \hline * & & \\ \hline & & * * \\ \hline & & * \\ \hline \end{array}.$$

Then the intersection of Oct_1 with $O(1, 2)$ and with $O(3, 4)$ defines a partition of $O(1, 2)$ and $O(3, 4)$, and gives us four of the tetrads we seek, namely

$$Oct_1 \cap O(1, 2), \quad O(1, 2) \setminus Oct_1, \quad Oct_1 \cap O(3, 4), \quad O(3, 4) \setminus Oct_1.$$

We now repeat the above process, slightly modified, this time with the aim of finding an octad contained in $D \cup (t_2 \cap Oct_1) \cup t_5 \cup t_6$ which itself contains $D \cup (t_2 \cap Oct_1)$. Using the MOG again we find

$$Oct_2 := \begin{array}{|c|c|c|} \hline & & * * \\ \hline & * & * * \\ \hline & & * * \\ \hline & * & * \\ \hline \end{array}.$$

Then $Oct_2 \cap O(5, 6)$ and $O(5, 6) \setminus Oct_2$ supplies the remaining two tetrads we need to define the sextet

$$Sex_1 := \begin{array}{|c|c|c|c|} \hline \diamond & * & * & + \\ \hline * & \times & \circ & - \\ \hline \diamond & - & - & + \\ \hline - & \times & \circ & * \\ \hline \end{array}.$$

REMARK. For each of Oct_1 and Oct_2 there is one other octad which also fulfils our requirements – however they will yield the same sextet. Also note that when finding Oct_1 we only needed to look in a certain 14-element subset of Ω and when finding Oct_2 in a certain 12-element subset of Ω . This, of course, makes our task easier.

Having pinpointed one sextet of the desired kind we enact the same procedure as above but starting with a 2-element subset D_1 of a tetrad of $S(c_{10})$ which is *not* contained in a tetrad of Sex_1 . So we may select D_1 to be

$$\begin{array}{|c|c|c|} \hline & & \\ \hline & * & * \\ \hline & & \\ \hline \end{array}.$$

Then, with the aid of the MOG, we find that

$$Oct_3 := \begin{array}{|c|c|c|} \hline & * & \\ \hline * & & \\ \hline & * & * \\ \hline & * & * \\ \hline \end{array}$$

is an octad contained in $D_1 \cup t_1 \cup t_3 \cup t_4$ which contains D_1 . And then

$$Oct_4 := \begin{array}{|c|c|c|} \hline * & & \\ \hline & * & * \\ \hline & * & * \\ \hline & * & * \\ \hline \end{array}$$

is an octad contained in $D_1 \cup (t_2 \cap Oct_3) \cup t_5 \cup t_6$ which contains $D_1 \cup (t_2 \cap Oct_3)$. Using Oct_3 and Oct_4 we obtain the following sextet

$$Sex_2 := \begin{array}{|c|c|c|c|} \hline \circ & \times & * & - & \diamond & * \\ \hline \times & * & \circ & - & \circ & \diamond \\ \hline \diamond & + & + & + & - & \diamond \\ \hline - & \times & \circ & * & \times & + \\ \hline \end{array}$$

A further sextet, Sex_3 , is obtained by taking the symmetric differences of the tetrads of Sex_1 and Sex_2 .

$$Sex_3 := \begin{array}{|c|c|c|c|c|c|} \hline \circ & \times & \diamond & + & \circ & \times \\ \hline \times & \times & * & - & \circ & \circ \\ \hline * & + & + & - & + & * \\ \hline - & \diamond & * & \diamond & \diamond & - \\ \hline \end{array}$$

By taking the symmetric differences of the tetrads of $S(c_{10})$ and Sex_1 , $S(c_{10})$ and Sex_2 , $S(c_{10})$ and Sex_3 we produce the three remaining sextets of the required types. To summarize, the chambers which are neighbours of c_{10} are

(3.3)

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$12|34|56$

$12|34|56$

$12|35|46$

$12|36|45$

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$12|34|56$

$12|34|56$

$12|34|56$

$12|34|56$

and

6	4	4	1	6	3
3	4	5	2	5	5
6	1	2	2	2	5
1	3	6	3	4	1

12|34|56

5	4	4	2	6	4
3	3	5	2	6	5
5	1	2	1	1	6
1	4	6	3	3	2

12|34|56

Finally we note that by resorting the data in (3.2) we obtain the disc sizes stated in Theorem 1.

4. Properties of \mathcal{C}

Though (3.2) presents \mathcal{C} from the viewpoint of c_0 , it is an easy matter to translate this information to another chamber c'_0 . Then the first, third and last two columns would, respectively, give $G_{c'_0}$ -orbits, distance between c'_0 and c , $\tau_i(T(c'_0))$ and $|O(c'_0) \cap O(c)|$ (c a chamber of \mathcal{C}). So, for example, we may use (3.2) to discover the distance between two given chambers c' , c'' . Our elimination procedure being as follows:

- (a) find i such that $S(c'') \in \sigma_i(S(c'))$;
- (b) using the $G_{c'}$ -orbits (obtained by suitably modifying (3.1)) find n such that $S(c'') \in \sigma_i^{(n)}(S(c'))$;
- (c) narrow down the possibilities in the $\sigma_i^{(n)}$ table, using $T(c'') \in \tau_k(T(c'))$ and $|O(c') \cap O(c'')|$; and finally
- (d) by examining the remaining options deduce to which $G_{c'}$ -orbit c'' belongs and thence obtain the distance between c' and c'' .

We give a 'real life' illustration, taking our chambers to be

$$c' = \begin{array}{|c|c|c|c|} \hline 1 & 2 & 2 & 2 & 1 & 4 \\ \hline 6^- & 3^+ & 4 & 5 & 5 & 1 \\ \hline 3 & 4^+ & 6 & 5 & 3 & 2 \\ \hline 5^- & 3 & 6 & 4 & 6 & 1 \\ \hline \end{array}, \quad c'' = \begin{array}{|c|c|c|c|} \hline 5^- & 4^+ & 3 & 4 & 4 & 2 \\ \hline 5 & 3^+ & 6 & 6 & 3 & 2 \\ \hline 1 & 5 & 3 & 1 & 2 & 1 \\ \hline 6^- & 2 & 1 & 4 & 5 & 6 \\ \hline \end{array} :$$

- (a) $S(c'') \in \sigma_0(S(c'))$;
- (b) (Here, of course, rather than mixed cols we should speak of mixed tetrads, where the tetrads are those of $S(c')$.) The mixed tetrads for $S(c'')$ are

$$\begin{array}{|c|c|c|c|} \hline 1 & & & 1 \\ \hline & & & 1 \\ \hline & & & 1 \\ \hline \end{array} \quad \text{and} \quad \begin{array}{|c|c|c|c|} \hline & & & 4 \\ \hline & 4 & & \\ \hline 4 & & & \\ \hline & & 4 & \\ \hline \end{array} .$$

Since the first of these tetrads is in

$$O(c') = \begin{array}{|c|c|c|c|} \hline * & * & * & * & * \\ \hline & & & & * \\ \hline & & & & * \\ \hline & & & & * \\ \hline \end{array}$$

and the other is not, we see that $S(c'') \in \alpha_0^{(768)}(S(c'))$.

- (c) Since $T(c'') \in \tau_0(T(c'))$ and $|O(c') \cap O(c'')| = 2$, one of the last eight possibilities of the $\sigma_0^{(768)}$ table in (3.2) must hold. Noting that $O(c') \cap O(c'')$ contains one element each from the two tetrads of $O(c')$ reduces the list of possibilities to four: $12|36|45$, $13|26|45$, $12|36|45$, $12|34|56$. (Incidentally, at this stage we know that c' and c'' are either distance 9 or 10 apart.)
- (d) We scrutinize the finer structure of the intersection of the octads of $T(c'')$ with $O(c')$. The octad of $T(c'')$,

	*	*	*	*
	*			*
		*		
			*	

intersects $O(c')$ in 4 elements which split 3|1 between the tetrads of $O(c')$, and the octad of $T(c'')$,

*				
*		*	*	
	*			
*			*	*

intersects $O(c')$ in two elements which are both contained in one of the tetrads of $O(c')$. Consequently, two possibilities remain, namely, $12|34|56$ and $13|26|45$.

The octad of $T(c')$, not equal to $O(c')$, which intersects $O(c'')$ in two elements is

			*	*
		*	*	*
*		*		*
*		*		*

and the intersection this octad has with $O(c'')$ is contained in one of the tetrads of $O(c'')$. So this rules out $13|26|45$ and therefore $12|34|56$ is the only possibility. Hence $d(c', c'') = 10$.

In fact, c' and c'' are both distance 10 from c_0 as the reader may verify using the above scheme.

We recall, from Section 3, that

$$c_{10} := \begin{array}{|c|c|c|c|c|c|} \hline 6 & 4 & 3 & 2 & 5 & 4 \\ \hline 3 & 3 & 5 & 1 & 5 & 6 \\ \hline 5 & 1 & 2 & 2 & 1 & 6 \\ \hline 2 & 3 & 6 & 4 & 4 & 1 \\ \hline \end{array} .$$

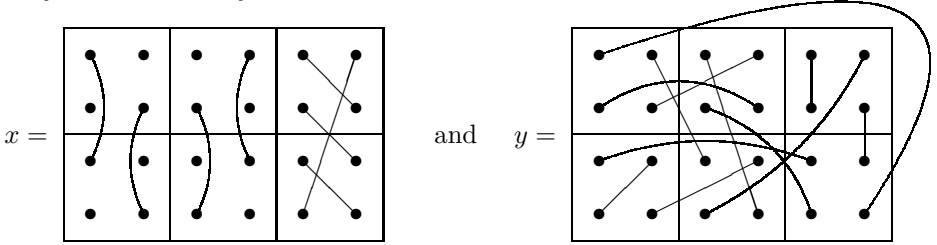
$12|34|56$

By (3.2) $c_{10} \in D_{10}(c_0)$, (c_{10} is the representative of the B -orbit given in the $\sigma_0^{(768)}$ table). Before analysing the geodesic closure of c_0 and c_{10} , we note that (3.2) implies that $B = G_{c_0}$ is transitive on $D_{10}(c_{10})$ and, combining (3.2) and (3.3), that c_{10} has no neighbours in $D_{10}(c_0)$. This establishes Theorem 2. Turning to Theorem 3, the following basic observation enables us, using (3.2), to enumerate the chambers

in $\overline{\{c_0, c_{10}\}}$: for j with $0 \leq j \leq 10$,

$$\overline{\{c_0, c_{10}\}} \cap D_j(c_0) = \overline{\{c_0, c_{10}\}} \cap D_{10-j}(c_{10}) = D_j(c_0) \cap D_{10-j}(c_{10}).$$

Set $\Delta_j = \overline{\{c_0, c_{10}\}} \cap D_j(c_0)$. Put



Then $x, y \in G$ (this may be checked using [5]) and $x^2 = y^2 = 1$. (We point out that x may be described by the octad it fixes point-wise and any one of the pairs of points it interchanges. Whereas y is an example of an involution given by the recipe: leave invariant the tetrads of a sextet and a dodecad which cuts across the sextet in 2^6 .) Moreover, x fixes c_0 and c_{10} (so $B_{c_{10}} = \langle x \rangle$) and y interchanges c_0 and c_{10} . Thus y interchanges the sets Δ_j and Δ_{10-j} for $0 \leq j \leq 10$. In Appendix A we list the chambers in $\overline{\{c_0, c_{10}\}}$. There we label each chamber by $c_j^k - j$ indicates that c_j^k is in the j th disc of c_0 (that is $c_j^k \in D_j(c_0)$) and the k is just a superscript. Our notation is arranged so as, when $j \neq 5$, y maps c_k^j to c_k^{10-j} . The existence of y means that it is only necessary for us to draw just over ‘half’ of $\overline{\{c_0, c_{10}\}}$; this we do in Appendix B.

To complete the story of the action of y upon $\overline{\{c_0, c_{10}\}}$ we have

(4.1) The orbits of y upon Δ_5 are $\{c_5^1, c_5^{14}\}$, $\{c_5^2, c_5^3\}$, $\{c_5^4, c_5^5\}$, $\{c_5^6, c_5^9\}$, $\{c_5^7, c_5^8\}$, $\{c_5^{10}, c_5^{13}\}$, $\{c_5^{11}, c_5^{12}\}$.

REMARK. y fixes no chambers in $\overline{\{c_0, c_{10}\}}$.

We also document the action of x upon $\overline{\{c_0, c_{10}\}}$. Since x and y commute, we only need consider Δ_i for $1 \leq i \leq 5$.

(4.2) The orbits of x upon Δ_i are as follows:

- $i = 1$: $\{c_1^1\}$, $\{c_1^6\}$, $\{c_1^9\}$, $\{c_1^{10}\}$, $\{c_1^2, c_1^4\}$, $\{c_1^3, c_1^5\}$, $\{c_1^7, c_1^8\}$
- $i = 2$: $\{c_2^1\}$, $\{c_2^6\}$, $\{c_2^7\}$, $\{c_2^{12}\}$, $\{c_2^2, c_2^4\}$, $\{c_2^3, c_2^5\}$, $\{c_2^8, c_2^{10}\}$, $\{c_2^9, c_2^{11}\}$
- $i = 3$: $\{c_3^1\}$, $\{c_3^8\}$, $\{c_3^9\}$, $\{c_3^{10}\}$, $\{c_3^{13}\}$, $\{c_3^{14}\}$, $\{c_3^2, c_3^5\}$, $\{c_3^3, c_3^6\}$, $\{c_3^4, c_3^7\}$, $\{c_3^{11}, c_3^{12}\}$
- $i = 4$: $\{c_4^1\}$, $\{c_4^2\}$, $\{c_4^9\}$, $\{c_4^{10}\}$, $\{c_4^{11}\}$, $\{c_4^{12}\}$, $\{c_4^{15}\}$, $\{c_4^3, c_4^6\}$, $\{c_4^4, c_4^7\}$, $\{c_4^5, c_4^8\}$, $\{c_4^{13}, c_4^{14}\}$
- $i = 5$: $\{c_5^1\}$, $\{c_5^6\}$, $\{c_5^7\}$, $\{c_5^8\}$, $\{c_5^9\}$, $\{c_5^{14}\}$, $\{c_5^2, c_5^4\}$, $\{c_5^3, c_5^5\}$, $\{c_5^{10}, c_5^{12}\}$, $\{c_5^{11}, c_5^{13}\}$.

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Δ_9 :

$$\begin{array}{l}
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 \\
 c_9^4 = \begin{array}{|c|c|c|c|c|c|} \hline 6 & 4 & 4 & 1 & 6 & 3 \\ \hline 3 & 4 & 5 & 2 & 5 & 5 \\ \hline 6 & 1 & 2 & 2 & 2 & 5 \\ \hline 1 & 3 & 6 & 3 & 4 & 1 \\ \hline \end{array} \\
 \\
 c_9^7 = \begin{array}{|c|c|c|c|c|c|} \hline 5 & 6 & 3 & 2 & 4 & 6 \\ \hline 3 & 3 & 4 & 1 & 4 & 5 \\ \hline 4 & 1 & 2 & 2 & 1 & 5 \\ \hline 2 & 3 & 5 & 6 & 6 & 1 \\ \hline \end{array} \\
 \\
 c_9^{10} = \begin{array}{|c|c|c|c|c|c|} \hline 2 & 4 & 3 & 5 & 1 & 4 \\ \hline 3 & 3 & 1 & 6 & 1 & 2 \\ \hline 1 & 6 & 5 & 5 & 6 & 2 \\ \hline 5 & 3 & 2 & 4 & 4 & 6 \\ \hline \end{array} \\
 \\
 c_9^2 = \begin{array}{|c|c|c|c|c|c|} \hline 5 & 4 & 4 & 1 & 6 & 4 \\ \hline 3 & 3 & 5 & 1 & 6 & 5 \\ \hline 5 & 2 & 1 & 2 & 2 & 6 \\ \hline 2 & 4 & 6 & 3 & 3 & 1 \\ \hline \end{array} \\
 \\
 c_9^5 = \begin{array}{|c|c|c|c|c|c|} \hline 6 & 3 & 4 & 2 & 5 & 4 \\ \hline 3 & 4 & 6 & 2 & 6 & 5 \\ \hline 5 & 1 & 1 & 1 & 2 & 5 \\ \hline 2 & 3 & 6 & 4 & 3 & 1 \\ \hline \end{array} \\
 \\
 c_9^8 = \begin{array}{|c|c|c|c|c|c|} \hline 4 & 5 & 3 & 2 & 6 & 5 \\ \hline 3 & 3 & 6 & 1 & 6 & 4 \\ \hline 6 & 1 & 2 & 2 & 1 & 4 \\ \hline 2 & 3 & 4 & 5 & 5 & 1 \\ \hline \end{array} \\
 \\
 c_9^3 = \begin{array}{|c|c|c|c|c|c|} \hline 6 & 3 & 4 & 2 & 6 & 3 \\ \hline 3 & 3 & 5 & 1 & 6 & 6 \\ \hline 5 & 2 & 2 & 1 & 2 & 5 \\ \hline 1 & 4 & 5 & 4 & 4 & 1 \\ \hline \end{array} \\
 \\
 c_9^6 = \begin{array}{|c|c|c|c|c|c|} \hline 5 & 3 & 3 & 1 & 6 & 4 \\ \hline 3 & 4 & 6 & 2 & 5 & 6 \\ \hline 5 & 2 & 2 & 1 & 1 & 5 \\ \hline 2 & 4 & 6 & 3 & 4 & 1 \\ \hline \end{array} \\
 \\
 c_9^9 = \begin{array}{|c|c|c|c|c|c|} \hline 4 & 2 & 1 & 6 & 3 & 2 \\ \hline 1 & 1 & 3 & 5 & 3 & 4 \\ \hline 3 & 5 & 6 & 6 & 5 & 4 \\ \hline 6 & 1 & 4 & 2 & 2 & 5 \\ \hline \end{array}
 \end{array}$$

Δ_{10} :

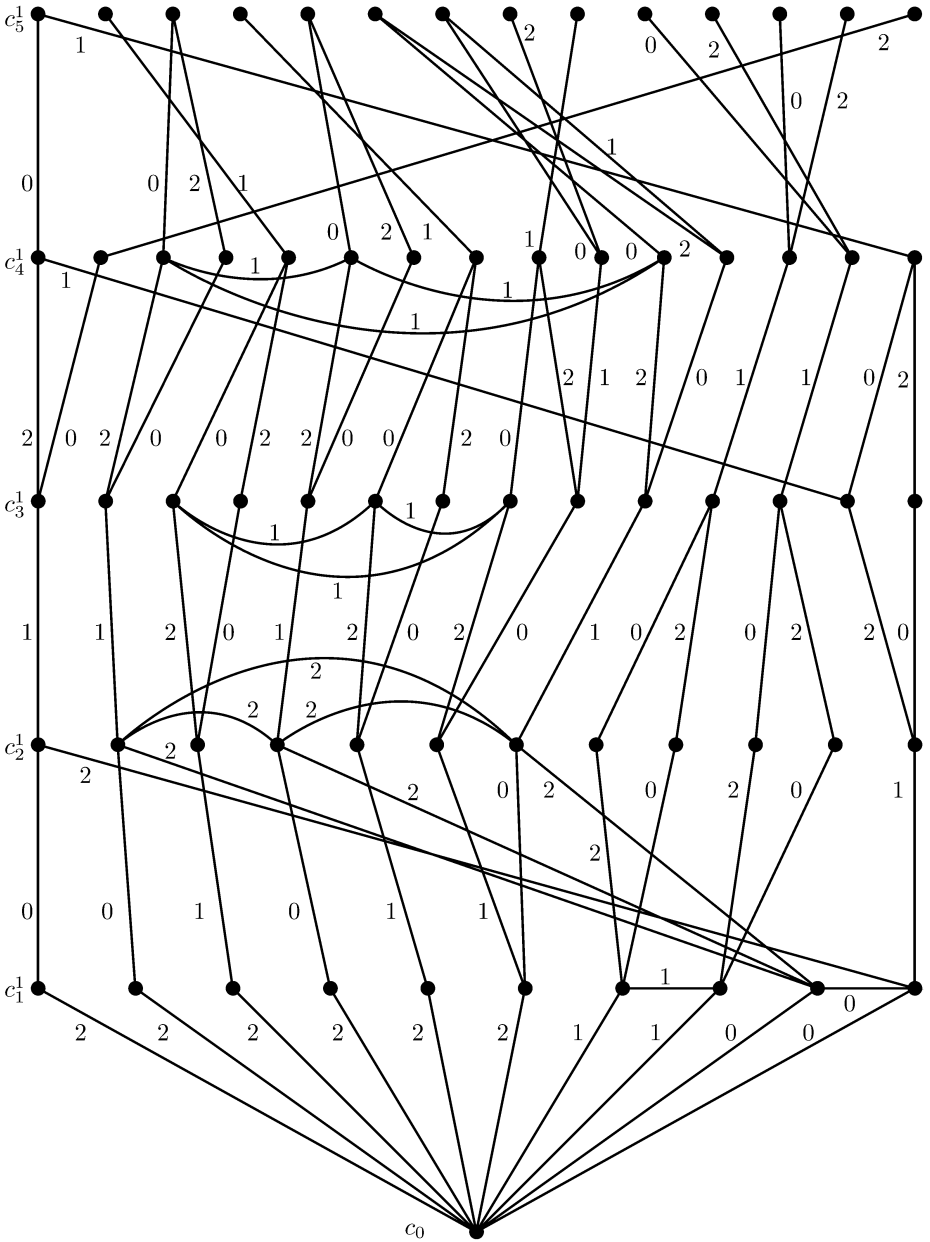
$$c_{10} = \begin{array}{|c|c|c|c|c|c|} \hline 6 & 4 & 3 & 2 & 5 & 4 \\ \hline 3 & 3 & 5 & 1 & 5 & 6 \\ \hline 5 & 1 & 2 & 2 & 1 & 6 \\ \hline 2 & 3 & 6 & 4 & 4 & 1 \\ \hline \end{array}$$

Appendix B.

On the next page we draw the induced subgraph of $\overline{\{c_0, c_{10}\}}$, together with the i -adjacency information. Recall that two adjacent chamber c' and c'' are said to be i -adjacent if $\tau(c') = \{i\} \cup \tau(c' \cap c'') = \tau(c'')$. So 0-, 1-, 2-adjacency means that c' and c'' ‘differ’ only in, respectively, an octad, a trio, a sextet.

As mentioned earlier we only need describe $\{c_0\} \cup \Delta_1 \cup \Delta_2 \cup \Delta_3 \cup \Delta_4 \cup \Delta_5 - \overline{\{c_0, c_{10}\}}$ can then be constructed from this using y .

So as not to clutter up our picture, the names of the chambers have been suppressed, with the exception of c_0 and c_j^1 ($1 \leq j \leq 5$). Reading downwards, in order, from c_j^1 gives the chambers c_j^k in Δ_j . Since $c_0, c_1^1, c_1^2, c_1^3, c_1^4, c_1^5$ and c_1^6 have the same octad and trio, any two of them are 2-adjacent. On our picture we have not drawn the 2-adjacency between c_1^m and c_1^n ($1 \leq m < n \leq 6$). All other adjacencies within $\{c_0\} \cup \Delta_1 \cup \Delta_2 \cup \Delta_3 \cup \Delta_4 \cup \Delta_5$ are given.



Appendix C.

```

m24:=sub<Sym(24)|(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,
20,21,22,23), (24,23)(3,19)(6,15)(9,5)(11,1)(4,20)(16,14)(13,21)>;
#m24;
244823040
sextet:=sub<m24|Stabilizer(m24,{24,3,6,9}),m24.2>;
Index(m24,sextet);
1771
triopw:=Stabilizer(Stabilizer(m24,{22,18,8,12,2,10,17,7}),
{24,3,6,9,11,4,16,13});
flag:=sub<m24|triopw,m24.2> meet sextet;
Index(m24,flag);
79695
CT:=CosetTable(m24,flag);
flagp:=CosetTableToPermutationGroup(m24,CT);
oo:=Orbits(Stabilizer(flagp,1));
[#oo[i]:i in [1..106]];
[ 1, 2, 2, 4, 4, 6, 8, 8, 12, 12, 12, 16, 24, 24, 24, 24, 24, 48,
48, 48, 48, 48, 48, 48, 48, 96, 96, 96, 96, 96, 96, 96, 96, 96,
96, 192, 192, 192, 192, 192, 192, 192, 192, 192, 192, 192, 192,
192, 384, 384, 384, 384, 384, 384, 384, 384, 384, 384, 384, 384,
384, 384, 384, 768, 768, 768, 768, 768, 768, 768, 768, 768, 768,
768, 768, 768, 768, 768, 1536, 1536, 1536, 1536, 1536, 1536,
1536, 1536, 1536, 1536, 1536, 1536, 1536, 1536, 1536, 1536,
1536, 3072, 3072, 3072, 3072, 3072, 3072, 3072, 3072, 3072 ]

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